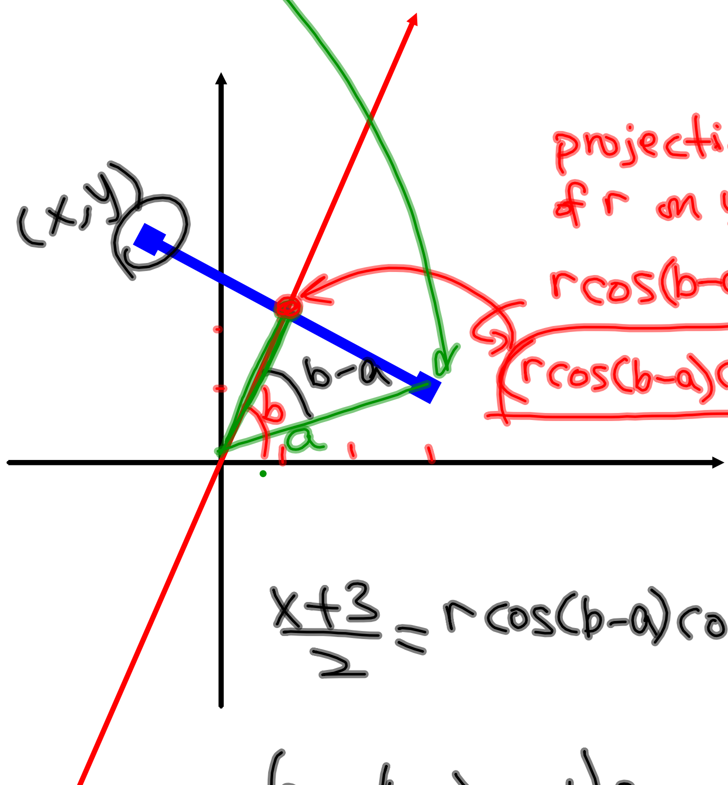
When (3,1) is reflected about *y* = 2*x*, find its image.



$r = \sqrt{10}$

$r\cos a = 3$

$r\sin a = 1$

projection of r on y=2x

$r\cos(b-a)$

$(r\cos(b-a)\cos b,\ r\cos(b-a)\sin b)$

$\tan a = \frac{1}{3}$

$18.43^\circ = a = \tan^{-1}\frac{1}{3}$

$63.43^\circ = b = \tan^{-1}\frac{2}{1}$

$$\frac{x+3}{2} = r\cos(b-a)\cos b$$

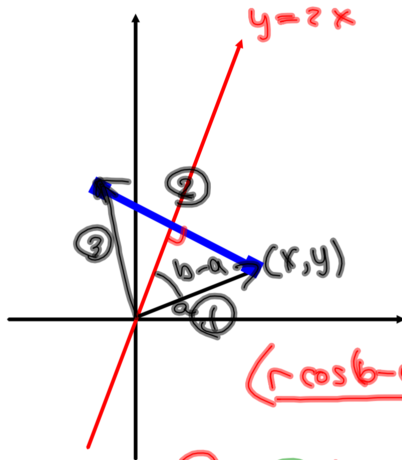
$$x = (r\cos(b-a)\cos b)2 \quad -3$$

$$= -1.00$$

$$\frac{y+1}{2} = r\cos(b-a)\sin b$$

$$y = 2r\cos(b-a)\sin b \ -1$$

$$\simeq 3.00$$



① + ② = ③

① $\langle x, y\rangle$

$(r\cos(b-a)\cos b,\ r\cos(b-a)\sin b)$

② $= 2\langle r\cos(b-a)\cos b - x,\ r\cos(b-a)\sin b - y\rangle$

① + ②
=

$\langle 2r\cos(b-a)\cos b - x,\ 2r\cos(b-a)\sin b - y\rangle$

$2r(\cos b\cos a + \sin b\sin a)\cos b - r\cos a$

$r(2\cos^2 b\cos a + 2\sin b\cos b\sin a - \cos a)$

$r\cos a(2\cos^2 b - 1) + r\sin a\ 2\sin b\cos b$

$(x\cos(2b) + y\sin(2b),\ \ x\sin(2b) - y\cos(2b))$

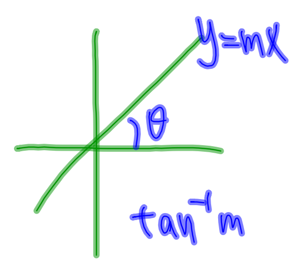
y: $2r\cos(b-a)\sin b - r\sin a$

$r(2(\cos b\cos a + \sin b\sin a)\sin b - \sin a)$

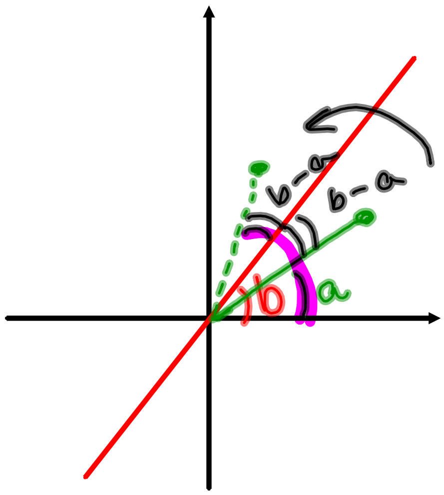
$2r\sin b\cos b\cos a + 2r\sin^2 b\sin a - r\sin a\cdot 1$

$x\sin(2b) \qquad - y\cos(2b)$

$$\begin{bmatrix}\cos 2b & \sin 2b\\ \sin 2b & -\cos 2b\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} =$$



$\tan^{-1} m$



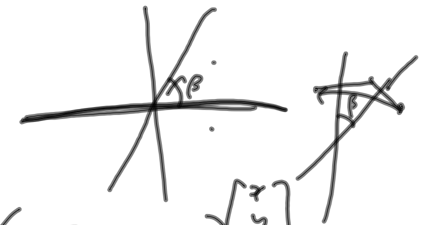
$a + b - a + b - a = 2b - a$

$(r\cos a, r\sin a)$

$(r\cos(2b-a), r\sin(2b-a))$

$= (r\cos 2b\cos a + r\sin 2b\sin a, r\sin 2b\cos a - r\cos 2b\sin a)$

$= (x\cos 2b + y\sin 2b, x\sin 2b - y\cos 2b)$

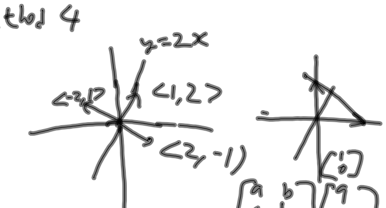


$\left(\left[2\beta\right]\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\right)\begin{bmatrix}x\\ y\end{bmatrix}$

$\begin{bmatrix}\cos 2\beta & -\sin 2\beta\\ \sin 2\beta & \cos 2\beta\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

$\begin{bmatrix}\cos 2\beta & \sin 2\beta\\ \sin 2\beta & -\cos 2\beta\end{bmatrix}$

method 4



$\begin{bmatrix}1\\ 0\end{bmatrix}$

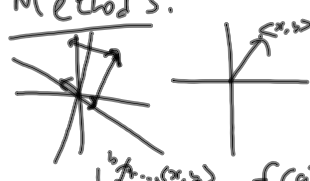
$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}a\\ c\end{bmatrix}$

$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}$ $\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}\begin{bmatrix}b\\ d\end{bmatrix}$

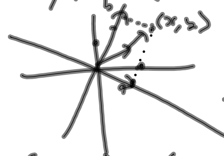
$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix}=\begin{bmatrix}-2\\ 1\end{bmatrix}$

$a+2b=1$ | $c+2d=2$
$2a-b=-2$ | $2c-d=1$

Method 5:



$\begin{bmatrix} & \end{bmatrix}$



$f(a)+f(b)=f(a+b)$
$k\,f(a)=f(ka)$

(2,-1) (1,2)
<1,0> <0,1>

$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$
$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}q\end{bmatrix}$

$\boxed{[G]^{-1}[R][G]\begin{bmatrix}x\\ y\end{bmatrix}}$

$a\,a=1$

$g^{-1}a\,g\;2\;a^{-1}a\,g$

What matrix has the effect of rotating every vector through 90° and then projecting the result onto the x-axis? What matrix represents projection onto the x-axis followed by projection onto the y-axis?

The matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ yields a ***shearing*** transformation, which leaves the y-axis unchanged. Sketch its effect on the x-axis, by indicating what happens to $(1,0)$ and $(2,0)$ and $(-1,0)$—and how the whole axis is transformed.